18th AMSE
APPLICATIONS OF MATHEMATICS AND STATISTICS IN ECONOMICS
International Scientific Conference | Jindřichův Hradec, 2-6 September 2015

PENSION SYSTEM FINANCING SCHEMES IN THE LIGHT OF THE AARON-SAMUELSON CONDITION

VLASTIMIL FARKAŠOVSKÝ
Matej Bel University, Faculty of Economics, Department of Finance and accounting,
Tajovského 10, 975 90 Banská Bystrica, Slovakia
email: vlastimil.farkasovsky@umb.sk

Abstract

Before 10 years, Slovak republic introduce reform of its pension system. From the system based on intergenerational solidarity, half of pension contributions was transferred to the private accounts managed by Pension management companies (PMC´s). Now, 10 years after reform, one of the most debated question in the Slovak republic is question of weak performance of the PMC´s. The paper in the first part deals with the long-term forecast of implicit performance of the Pay-as-you-go pension scheme in the Slovakia in the light of the Aaron-Samuelson condition derived from the simple overlapping generations model (OLG model). In the second chapter, the performance of the capitalization pillar of the Slovak pension system is calculated and results of both computation are compared and used for evaluation of the benefit of introducing capitalization pension scheme to the Slovak pension system

Key words: *Aaron-Samuelson condition, implicit rate of return, pension funds profitability.*

Introduction

Demographic trends having a significant impact on the sustainability of the pension system and also has a significant impact on the number of workforce in the economy. Decreasing mortality and natality rate collectively referred as a population ageing will create a huge future pressure on public finances and financing future pension benefits. These hidden liabilities in the form of implicit debt can create a serious trouble reinforced in the light of current debt crisis. Several countries, including the Slovak Republic has therefore opted for the introduction of the capitalization pillar of the pension system, which should be the solution to these problems.

The main aim of the paper is therefore verifying the validity of Aaron-Samuelson condition in the Slovak Republic and in the light of this theorem assessing the appropriateness of introducing the capitalization pillar of the pension system in the Slovak republic. For analyzing we choose OLG model as

The contribution is divided to 5 parts from which the first one is introductory and the last one is concluding. The paper in its second part describes Aaron-Samuelson condition, concept of implicit debt and implicit profitability of the PAYG (Pay-as-you-go) pension pillar. In the third part used data sets and paper methodology are described. Contribution also deals with some specifications of used overlapping generations model connected with the problematics of Aaron-Samuelson condition. The fourth part is devoted to results of computing and assess the appropriateness of introducing the capitalization pillar in the Slovak republic.

1. Aaron-Samuelson condition

Samuelson (1958) in his work proved by a simple model of two overlapping generations (Overlapping generations model - OLG) that the cost of ongoing transformation of the pension system to a fully funded system is equal to the amount of contributions that were paid on the income of the first generation of pensioners (implicit debt). Implicit liabilities are under Constitutional law no. 493/2011 Coll. about Fiscal Responsibility defined as "the difference between the expected future expenditure of general government and the future revenue of government entities resulting from the financial consequences of a future exercise of the rights and obligations established by Slovak law, unless they are part of government debt". This definition is constructed for net implicit debt, which involves not only future expenditures but also future incomes. According to Ministry of Labour, Social Affairs and Family of the Slovak republic (2007), implicit debt of the pension system at 2003 was at level of 300% GDP. The latest general government budget notes fall of implicit debt to the level of 229.2% of GDP in 2014. This change was driven mainly by pension reform. Pension reform establish capitalization pillar of pension system by transferring the part of contribution from PAYG system to private accounts in Pension management companies (PMC´s). Transformation of implicit debt to explicit debt is manifested precisely in the transformation costs of the pension reform and the concomitant increase in the deficit of the Social Insurance Agency (SIA), which resulted to the government response in form of decreasing the contributions to a capitalization scheme from 9% to 4% with the promise of a gradual increase to 6% between 2017-2024.

It must be emphasized that the productive generation contributory creates a liability for future generations, the amount of such liabilities may change under the influence of inflation and changes in labor productivity or demographic development. It actually represents a return of PAYG scheme (implicit rate of return). This facts, together with the seminal work of Aaron led to the formulation of Aaron-Samuelson rule. Andersen and Bhattacharya (2012) define it as a simple, by calculations based on dynamic system of differential equations proven and potentially verifiable condition, which says that the pay-as-you-go pillar of the pension scheme is socially desirable if and only if the return on capital is lower than the growth rate economy, respectively, in the case where the sum of productivity growth and workforce in the economy of the country (implicit return PAYG) exceeds the rate of recovery of assets of funded schemes (Queisser and Whitehouse, 2006). Rievajová (2011) notes that implicit rate of return of the first pillar at the same time setting the pace of wage growth in the economy.

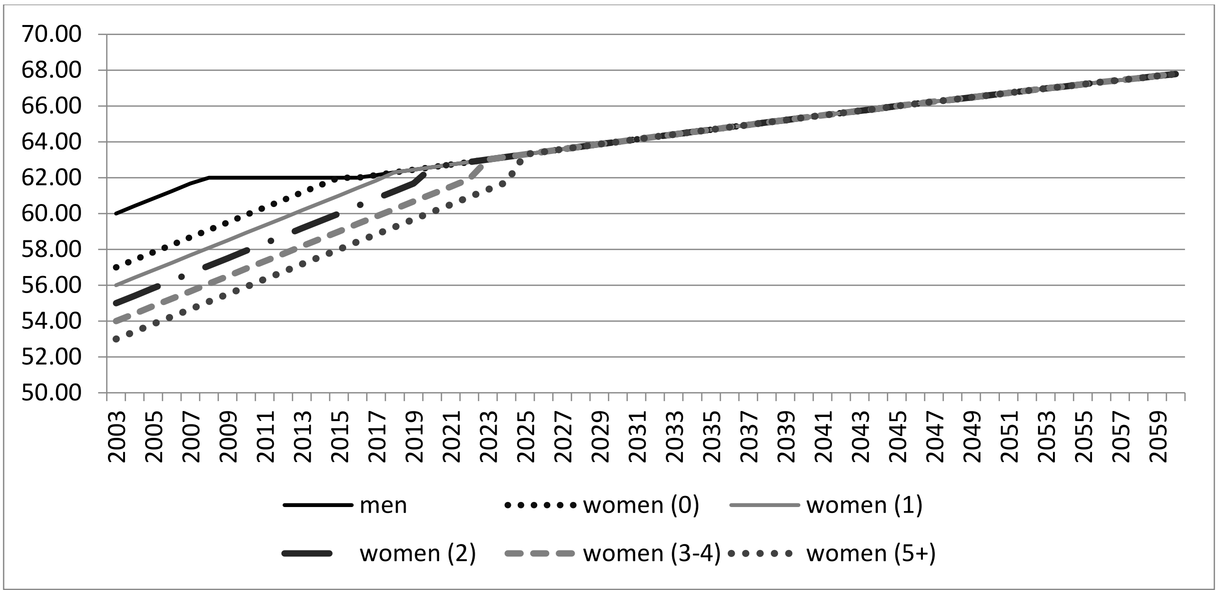
When we are looking on PAYG system only through above mentioned point of view for calculating future pension benefits, this argument is sufficient because, the benefit formula contains only inflation and wage change. However, when viewing the pension system in terms of income, it is clear that due to demographic shift this status must be reviewed. For these reasons, we suggest new calculations of implicit PAYG return considering not only the changes in labor productivity but also a changes in the share of the labor force to economically inactive part of the population.

By the verification of this modified Aaron-Samuelson condition we can show that the techniques of pension funding are more beneficial in terms of overall benefits and the contribution to the long-term sustainability. Long-term sustainability, however, does not necessarily mean even greater stability. The problem of stability is primarily associated with the second demographic transition as called by Hamerníková and Maatyová (2010).

2.1 Data and methodology

Over the time, OLG models became one of the prominent tools for government policy research. Hagemejer et al. (2015), Cipriani (2014), Fehr and Uhde (2013), Buyse at al. (2013) used it in their research of pension system. One reason is, that studies researching government policies for a long term period often find out, that the assumption of representative agent is not appropriate mainly due to infinite life horizon. In overlapping generations models new individuals are continually being born and dying and after every time shift a generations can have new socioeconomic characteristics.

Our model is partly based on that of de la Croix (2013). We consider an overlapping generations model with 13 generations in each period (seven productive age generations 25-65 and six between 65-85+). Matlab software and Dynare platform was used to compute system of linear and nonlinear difference equations to find the system steady state. Uncertainty and intra-generational heterogeneity are assumed away. The young supply their labor inelastically, receive labor wages and consume and/or save their disposable income. The old deplete and consume their savings and asset incomes. Demographic development is calculated as exogenous variable in accordance to Vaňo (2012) forecasts. Labour market is modelled according to DMP model. DMP model is an extension of Mortensen and Pissarides model by the contribution of P. Diamond. All authors for their contribution gained in the 2010 Sveriges Riksbank Prize in Economic Sciences in Memory of A. Nobel. Retirement age is calculated according to Slovak law which assume gradual increase shown in the graph 1.



Graph 1. Increasing the retirement age between 2004 and 2060 (number in bracket marks number of children).
Source: Own processing

The proportion of the working population (PET_t) on the pension beneficiaries (PPT_t) can be expressed as follows:

$$SH_t = \frac{PET_t}{PPT_t} \quad (1)$$

This figure is one of the forms of economic dependency indices. For calculations of the development of number of workers (PET_t) formula (2) can be applied:

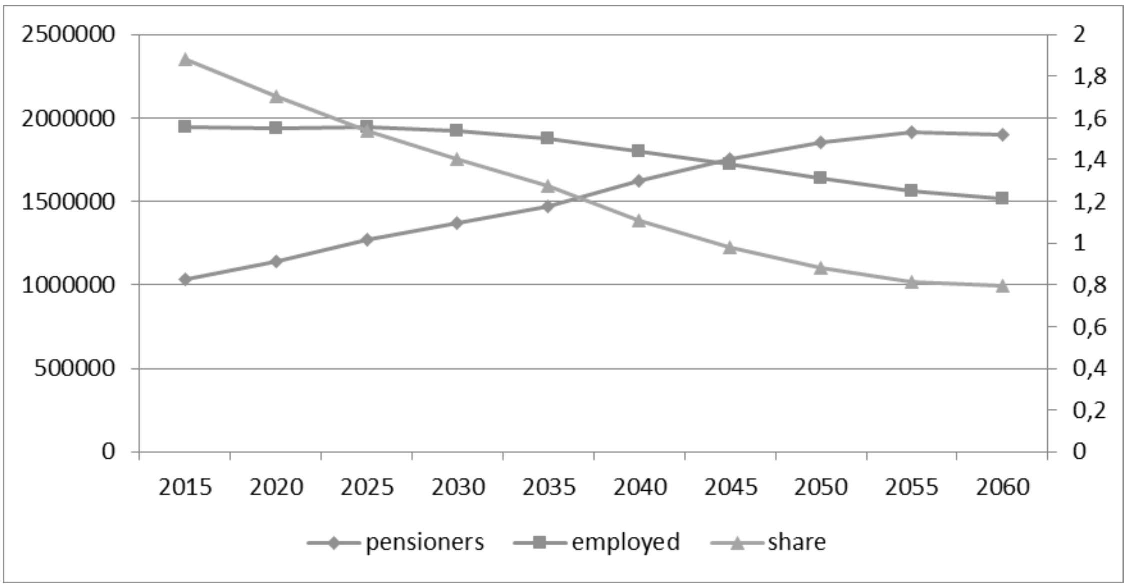
$$PET_t = e_t * PAT_t \quad (2)$$

Where e_t is the employment rate and PAT_t is the population in working age. Data for predicting population trends of the age groups are drawn from Vaňo (2012).

The average wage AW_t in the economy which determines amount of retirement benefits in the PAYG pension pillar is obtained as the following proportion, where $w_{a,t}$ is average wage at population group:

$$AW_t = \frac{\sum_{a=0}^{7} w_{a,t} * P_{a,t}}{PAT_t} \quad (3)$$

The development of this relationship is displayed in the chart no. 2 together with the development of the working population and the population in receipt of pension benefits. Development of the number of workers and residents in receipt of pension benefits shows the left vertical axis, while the ratio between the two groups is spotted on the right vertical axis.



Graph 2. Evolution of the number of workers, pension beneficiaries and their ratio
Source: Own processing based on data RRZ and the development of population by 2060

One of the most important blocks of the model describe development in the social security system. In the model we assume that old-age pension spending $\exp_t^e$ and spending's on early retirement $\exp_t^r$ represents a proportion of the average wage in the national economy. Their amount is then defined as follows:

$$\exp_t^e = \rho_t^e * AW_t * (w_{6,t} * \lambda_{6,t} * P_{6,t} + w_{7,t} * \lambda_{7,t} * P_{7,t}) \tag{4}$$

$$\exp_t^r = \rho_t^r * AW_t * (PT_t - PAT_t) \tag{5}$$

Where ρ_t^e is average early retirement replacement rate and ρ_t^r represents replacement rate in the old age. $\lambda_{a,t}$ stands for share of people in early retirement. Data on income replacement rates in retirement or early retirement was gained as a proportion of the average amount of retirement or early retirement pension to the average wage. Average amounts of old-age pensions are available in The management reports of the Social Insurance Agency (Správy o hospodárení Sociálnej poisťovne). w_t - the total wages in the national economy and the rate of levy-tax burden was included to calculation in the form of implicit taxation of labor. Total wages in the national economy can be expressed by the following: equation:

$$w_t = \sum_{a=0}^{7} w_{a,t} * e_{a,t} * P_{a,t} \tag{6}$$

Funded pension pillar profitability is calculated according to the more accurate modified IRR methodology concept described by Farkašovský (2014).

IRR can be computed according to the following formula:

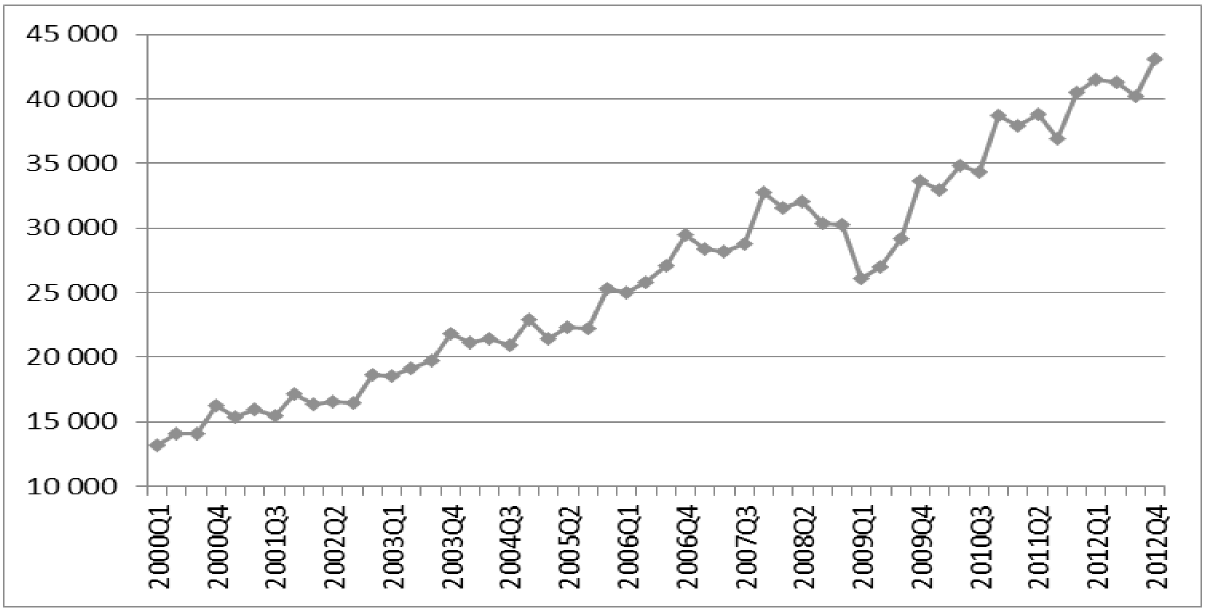
$$\sum_{k=0}^{K}\left(\left(NAV_{k+1} - NAV_k * \left(1 + \frac{PUCV_{k+1} - PUCV_k}{PUCV_k}\right)\right) * (1+IRR)^{\frac{k}{365}}\right) = 0 \tag{7}$$

PUCV – Weigted Pension unit current values aggregate representing the funded pillar PUCV
IRR - internal rate of return (equal to actual nominal rate of return of pension fund)
K – number of business days till period of computing
k – positive integer
NAV_k – Net asset value of fund at business day k
NAV_{k+1} – Net asset value of fund at business day k+1

In respect to this, the nominal internal rate of return of the funded pension pillar is computed by widely used iterative numerical methods using generalized reduced gradient algorithm for optimizing nonlinear problems. More information about methodology of computing can be found in (Lasdon et al., 1998). Accuracy of computing is set to 0.00001 %. In accordance to this methodology, the system of *K* (*K*=2250) nonlinear equation is solved. Daily data about *PUCV* and *NAV* were obtained from database of National Bank of Slovakia for period of 9 years since the establishment of PMCs 22$^{\text{th}}$ March 2005 till 21$^{\text{th}}$ March 2014, what represents 2250 business days.

2.2 Empirical results and discussion

Lisý (2007, pp. 380) argues that "the real GDP growth depends on labor productivity and the number of economically active population. Under labor productivity means a quantity of goods produced for some time "in this case attributable to one working. Quarterly labor productivity from sales of own products and goods in € for the period of 2000-2013 is shown in the following chart.



Graph 3. Quarterly labor productivity from sales of own products and goods in €
Source: Own processing based on data from the Statistical Office of the Slovak Republic

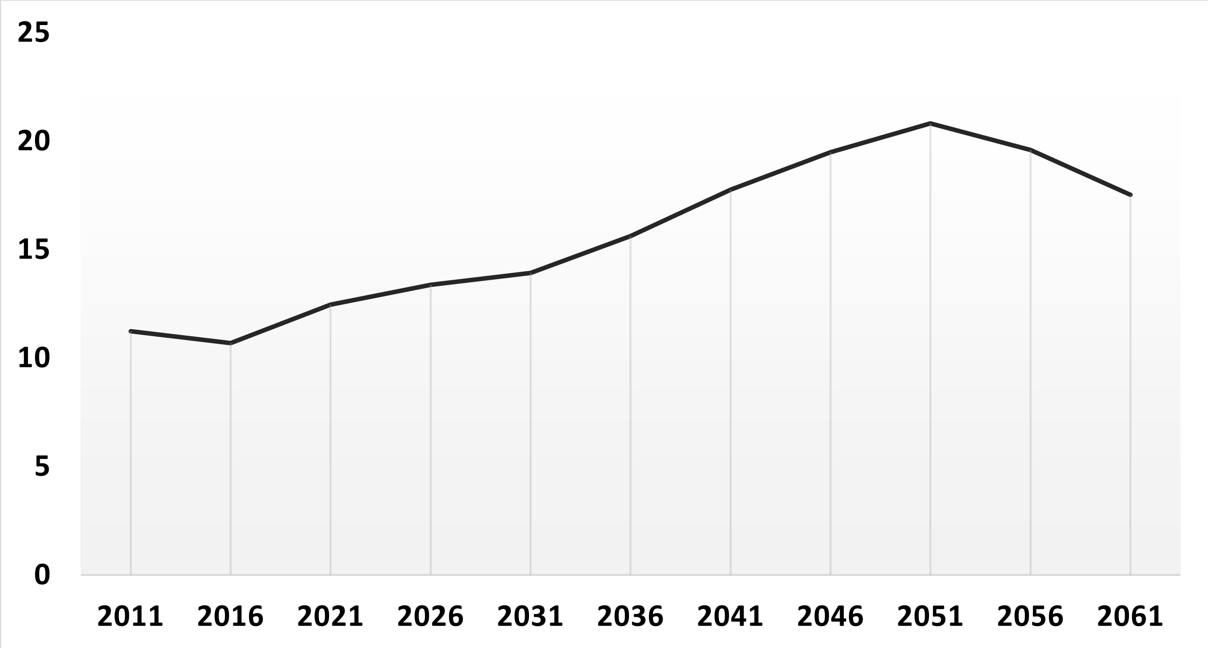
Like economic growth depending on the number of economically active population and their productivity, PAYG pension scheme is also dependent on these factors.

By solving of 13 generations OLG model based on the system of linear and nonlinear difference equations, the expected implicit rate of return of the PAYG pension scheme in the Slovak republic was calculated. It represents the first step in evaluating Aaron-Samuelson condition.

The average implicit rate of return of the PAYG pillar till year 2060 for the Slovak conditions was enumerated in the amount of plus 0.2754 % per annum.

Thus we see that virtually all the expected growth in the labor productivity is negated due to a decline of the number of workers on the number of persons receiving pension benefits and implicit PAYG return is very low.

Due to the increase in number pension beneficiaries and decrease of the economically active population, pension expenditures are expected to increase. Simulation of predicted changes between years 2011-2061 in the public expenditures on old-age pensions are displayed at a graph no. 4. According to this, public pension expenditures will double around 2050, when peaked a top a subsequently some decrease is expected.



Graph 4. OLG prediction of the amount of PAYG pillar expenditures as a % of GDP between years 2011-2061
Source: Own processing

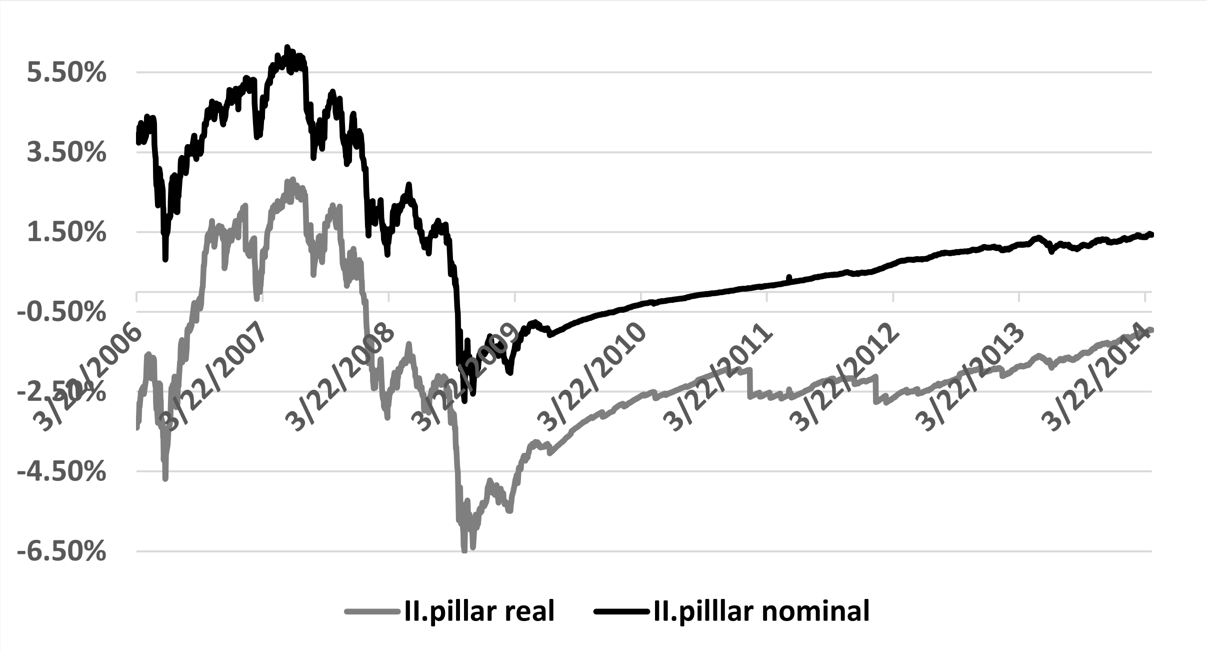
To be able to evaluate Aaron-Samuelson condition it is necessary to compute internal rate of return of the capitalization pension scheme. This return was calculated for the 9 years of existence of the capitalization pension pillar in the Slovak republic using modified IRR methodology described in previous chapter.

Profitability rate in the form of the funded pension pillar internal rate of return for the last 10 years is on the level of minus 1.89 % p.a.

Moreover since it is not resolved the issue of financing the deficit of the Social Insurance Agency due to revenue shortfalls caused transferring part of the contributions to the accounts of the pension fund management companies, the deficit must be covered from the state budget, the real return on funded pillar would still should be reduced by the amount of interest paid in order to ensure the financing of the resulting deficit.

Based on this information, assuming the validity of Aaron-Samuelson conditions appears to Slovak conditions favorable continuous process for funding future pensions.

At the following chart we can see real and nominal daily values of internal rate of return of the whole capitalization pillar in the Slovak republic computed to the beginning of pillar in % p.a. Maximum real rate of return of the capitalization pillar occurred at 18.6.2007, when we spotted real rate of return on the level of plus 2.82 % p.a. Following financial crisis decreased real rate of return to the minimum reached minus 6.51 % p.a. what means that every euro invested till this date was depreciated by more than 6.5 % every year. At the chart we can also see the legislative amendment after peak of the financial crisis, which was targeted for dealing with this situations by the form of guarantees resulting in very conservative investments of PMC´s. In this period we can see low, but extremely non-volatile development of capitalization pillar rate of return.



Graph 5. Real and nominal rate of return of funded pension pillar in the Slovak republic between 22.3.2006 – 22.3.2014 in % p.a.
Source: Own processing

Conclusion

The public pension scheme is very important due to its scope and impact on the lives of citizens rank among the most important elements of the social system of the Slovak Republic. This system is more susceptible to the aging population, which was also the reason for the introduction of the capitalization pillar of the pension insurance. Such a step, however, is in the light of the Aaron-Samuelson condition socially desirable if and only if the return on capital is higher than the growth rate of economy, respectively, in the case where the rate of recovery of assets of funded schemes exceeds the sum of productivity growth and workforce in the economy of the country (implicit return PAYG).

The main aim of the contribution was therefore verifying the validity of Aaron-Samuelson condition in the Slovak Republic and in the light of this theorem assessing the appropriateness of introducing the capitalization pillar of the pension system in the Slovak republic.

For computation of an implicit rate of return, the 13 generational OLG model was used to find a general equilibrium. To compute capitalization pillar rate of return, we used modified methodology based on internal rate of return to compute it nominal and real profitability.

The results indicates, that the implicit rate of return of PAYG pillar outstripped rate of return of the capitalization pillar, which means in the light of Aaron-Samuelson condition greater advantage of PAYG pillar compared to capitalization.

Original contribution of the work we see in the use of more advanced OLG model to calculate the implicit rate of return of the PAYG pillar and the modified calculation of the IRR, as well as the application of Aaron-Samuelson conditions in the Slovak Republic since the introduction of the theme of the benefit of II. pillar does not cover any publication now.

References

1. AARON, H. J. 1966. The Social Insurance Paradox. In Canadian Journal of Economics, vol. 32, 1966, pp. 371-374. ISSN 1540-5982.
2. Act no. 493/2011 Coll. on Fiscal Responsibility. [cit. 12-04-2014] http://www.nrsr.sk/web/Dynamic/Download.aspx?DocID=362187.
3. ANDERSEN, T., BHATTACHARYA, J. 2013. Unfunded pensions and endogenous labor supply. In Macroeconomic Dynamics. Cambridge : Cambridge University Press, 2013, pp. 1-27. ISSN 1469-8056.
4. BUYSE, T. et al. 2013. Pension reform, employment by age, and long run growth. In Journal of population economics, 2013, pp. 769-809. ISSN 0933-1433.
5. CIPRIANI, G. P. 2014 Population aging and PAYG pensions in the OLG model. In Journal of population economics, 2014, pp. 251-256. ISSN 0933-1433.
6. De la CROIX, D. et al. 2013. Aging and pensions in general equilibrium. In Journal of Economic Dynamics and Control. Columbia : University of Missouri, 2013, pp. 104-124. ISSN 0165-1889.
7. FARKAŠOVSKÝ, V. 2014. New Concept of Pension Funds Performance Evaluation. In Application of Mathematics and Statistics in Economy. 2014. Wroclaw : Wroclaw University of Economics, pp. 85-94. ISSN 1857-7881.
8. FEHR, H., UHDE, J. 2013. On the optimal design of pension systems. In Empirica, 2013, pp. 457-482. ISSN 0340-8744.
9. HAGEMEJER, J. et al. 2015. Unprivatizing the pension system: the case of Poland. In Applied economics, 2015, pp. 833-852. ISSN 0003-6846.
10. HAMERNÍKOVÁ B. et al. 2010. Veřejné finance. Praha : Wolters Kluwer, 2010. ISBN 978-80-7357-497-0.
11. LASDON, L. et al. 1998. Design and use of the Microsoft Excel Solver. In Interfaces, 1998, pp. 29-55. ISSN 0092-2102.
12. LISÝ, J. 2007. Ekonómia v novej ekonomike. Bratislava : IURA Edition, 2007, pp 634. ISBN 978-80-8078-164-4.
13. MINISTRY OF LABOUR, SOCIAL AFFAIRS AND FAMILY. 2007. Analýza vplyvu zavedenia II. kapitalizačného piliera na hospodárenie Sociálnej poisťovne. Bratislava : Ministerstvo práce, sociálnych vecí a rodiny, 2007. [cit. 13-04-2014] http://hsr.rokovania.sk/data/att/39998_subor.doc.
14. QUEISSER, M., WHITEHOUSE, E. 2006. Neutral or fair? Actuarial concepts and pension-system design. In OECD Social, employment and migration working papers. Paris : OECD Publication service, 2006, pp. 1-42. ISSN 1815-199X.
15. RIEVAJOVÁ, E. et al. 2011. Sociálne zabezpečenie. Bratislava : Vydavateľstvo Ekonóm, 2011. ISBN 978-80-225-3190-0.
16. SAMUELSON, P. 1958. An exact Consumption-Loan Model of Interest without the Social Contrivance of Money. In Journal of Political Economy. Chicago : The University of Chicago, 1958. ISSN 0022-3808.
17. Správy o hospodárení Sociálnej poisťovne za roky 2004-2013. [online]. Sociálna poisťovňa [cit. 20-03-2014]. http://www.socpoist.sk/ext_dok-4b-hospodarenie-iq/.
18. VAŇO, B. et al. 2012. Prognóza obyvateľstva SR do roku 2060. [online]. 2012. Bratislava : Výskumné demografické centrum INFOSTAT, 2012, pp. 74. [cit. 2013-02-22]. http://www.infostat.sk/vdc/pdf/obyv452000.pdf.